T. G. Keegan 12-17-7

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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE - "A Study of Two-Impulse Maneuvers for CSM Rescue of a Disabled LEM"

TM - 65-2013-1

FILING CASE NO(S) - 310

DATE - November 29, 1965

AUTHOR(S) - L. P. Gieseler

FILING SUBJECT(S) - (ASSIGNED BY AUTHOR(S) -
LEM Ascent
LEM Rescue
Abort Studies
Rendezvous Studies

ABSTRACT

This memorandum reports an extension of the work done by Hsu Lo and the writer during the summer of 1964, in which the requirements for the CSM rescue of a disabled LEM were studied. (See Reference 1) Initially a simplification is made by assuming that the rescue orbit will be collinear with the LEM orbit. Rendezvous will then take place either at the periselene or the aposelene of the LEM orbit, and the governing equations are consequently very simple. The velocity increment required by the CSM to effect the rescue is shown to be comparatively small.

It is then shown that the above mentioned maneuvers are a subset of the more general non-collinear rescue orbits whose CSM velocity requirements may exceed 680 fps. The results are shown as graphs which give the conditions under which a rescue operation is possible.

A list of symbols and their definition is given in Appendix I, and the equations used for the computation of non-collinear rescue orbits are given in Appendix II.

(NASA-CR-111071) A STUDY OF TWO-IMPULSE MANEUVERS FOR A CSM RESCUE OF A DISABLED LEM (Bellcomm, Inc.) 45 p

N79-72206

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00/16 11930

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SUBJECT: "A Study of Two-Impulse Maneuvers for CSM Rescue of a Disabled LEM" Case 310

DATE: November 29, 1965

FROM: L. P. Gieseler

TM-65-2013-1

TECHNICAL MEMORANDUM

A. INTRODUCTION

In the most general lunar surface abort situation, the LEM is launched from the lunar surface and attains some sort of safe orbit, but is incapable of further propulsion, and requires rescue by the CSM. A convenient parameter for categorizing the LEM orbit is ΔV_{LO}, the velocity excess at burn-out over nominal circular velocity at 50,000 feet altitude. For the nominal non-abort mission this velocity excess equals 99 fps and would result in a Hohmann transfer ellipse with periselene at a 50,000 feet altitude, and an aposelene at an 80 NM altitude. In this study the velocity excess was varied from -50 fps to +350 fps, and a determination was made of the relationship among the important orbital variables so that rescue may be examined over a wide range of conditions. The study is restricted to two-impulse two-dimensional rescue maneuvers.

B. CONCLUSIONS

For the assumed range of LEM velocity excess the velocity increment required by the CSM to rescue the LEM (ΔV_{CT}) varies from 99 to 350 fps if transfer orbits are used whose major axes are collinear with the major axis of the LEM. The higher

figure corresponds to the upper bound placed on the velocity excess and is not an absolute maximum. A requirement for collinear transfer is that the velocity excess and the timing of the LEM launch cannot be considered completely independent. For a given velocity excess only discrete launch opportunities are available. In general the number of discrete launch opportunities becomes greater as the maximum allowable time from LEM launch to rescue is increased.

Between the discrete collinear launch opportunities non-collinear orbits must be used. These were explored numerically for two cases for which the velocity excess equaled 51 and 273 fps. The timing of the LEM launch was varied continuously between values required for collinear rescue orbits. The results showed a marked difference between these two cases, the former giving a CSM rescue velocity increment almost equal to the corresponding collinear cases, and the latter indicating a substantial increase in the velocity increment. It is concluded that non-collinear rescue orbits may be used freely for small values of LEM velocity excess, but when it is of the order of 270 fps, the CSM rescue velocity increment requirement may exceed 680 fps.

C. SPECIFICATION OF A 2-IMPULSE RENDEZVOUS MANEUVER

It is convenient to regard the CSM and LEM as permanently located in their respective circular and elliptical orbits, and to postulate a third vehicle which is permanently in the transfer orbit. A rendezvous maneuver occurs when the CSM and transfer vehicle occupy the same position at one value of time, and the

transfer vehicle and LEM occupy the same position at a later time. There are three values of the time which are of interest:

1. $t = 0$. This is taken to be the time that the thrust of the LEM ceases. The angular positions of CSM, transfer vehicle and LEM are represented by θ_{CO}, θ_{TO}, and θ_{LO}. The corresponding radial distances are R_C(a constant), R_{TO}, and R_{LO}.

2. $t = t_A$. This is the time that the CSM and transfer vehicle occupy the same position. Angles and radial distances of CSM, transfer vehicle, and LEM are represented by θ_{CA}, R_C, θ_{TA}, R_{TA}, θ_{LA} and R_{LA} respectively.

3. $t = t_B$. This is the time that the transfer vehicle and the LEM occupy the same position, and rendezvous occurs. Angles and radial distances of CSM, transfer vehicle, and LEM are represented by θ_{CB}, R_C, θ_{TB}, R_{TB}, θ_{LB}, and R_{LB}, respectively. The time of flight in the transfer orbit is represented by t_f, and equals $t_B - t_A$.

It will now be shown that excluding the gravitational constant, there are seven independent quantities which need to be specified to define a rendezvous maneuver of the type shown in Figure 1.

CSM Orbit. This orbit is completely determined when the radial distance R_C, an angular location of the CSM, and the corresponding time are given. Let the last two quantities be θ_{CA} and t_A.

LEM Orbit. The shape of this orbit is determined by a_L and e_L. (See Appendix I for a list of symbols.) Its orientation is determined by θ_{LP}, the angular position of the periselene. The location of the vehicle in the orbit will be specified by assuming that when $t = 0$, $\theta_{LO} = 0$. This locates the reference line of Figure 1 through the position that the LEM occupies when $t = 0$.

Transfer Orbit. If t_B is specified, R_{LB} and θ_{LB} can be determined since the LEM orbit is known. R_C, θ_{CA}, and t_f are also known. The transfer orbit is then completely specified since two positions on the orbit, and the time required for the transfer vehicle to travel from one to the other are known.

The seven quantities R_C, θ_{CA}, t_A, a_L, e_L, θ_{LP}, and t_B will then completely specify the rendezvous maneuver. Quantities other than these may of course be used, as long as they are independent. Because of the cyclic nature of some of the governing equations, it is possible that one set of numerical values will specify more than one rendezvous maneuver, and conversely, the same maneuver may be specified by more than one set of values of the independent variables.

D. REDUCING THE NUMBER OF VARIABLES

For making a parametric study, it is desirable to restrict the above general problem in a number of ways so that the seven independent variables are reduced to two. One can then use these two quantities as coordinate variables, and plot other quantities of interest as equi-value contours. In this memorandum this reduction is done as follows:

1. R_C = constant, i.e., only one CSM circular orbit having an altitude of 80 NM will be considered.
2. Altitude of periselene of LEM orbit = 50,000 ft, and $\theta_{LP} = 0$. These restrictions require that at the end of powered flight the flight path angle will be zero. Thus the LEM orbit is defined by a single parameter, which is conveniently taken as ΔV_{LO}, the excess velocity over circular velocity that the LEM will have at powered flight cut-off.
3. Both t_A and t_B are specified by requiring that they have values which give minimum total CSM velocity increment (ΔV_{CT}) for effecting the rescue. Section F describes an exception to this, where a method is given for obtaining a non-optimum t_B. t_A alone is then optimized.

The two remaining independent variables will be taken as ΔV_{LO}, and θ_{CO}, the lead angle of the CSM when t = 0. The

latter quantity is easily varied in the actual maneuver by changing the timing of LEM launch.

E. COLLINEAR CASE

A great deal of insight into the rendezvous maneuver can be attained with very simple equations by considering first those cases where the axis of the transfer orbit is in line with the axis of the LEM orbit. As shown in Figures 2a and 2b, there are two cases to be considered, rendezvous at LEM orbit periselene, and rendezvous 180° from it. In each case the problem is to determine from various given values of θ_{CO} what the characteristics of the transfer maneuver are. It is more convenient, however, to specify various values of ΔV_{LO}. This then determines the transfer orbit, from which θ_{CO} can be calculated. Values for some of the angles can be written down by inspection, and are shown in Table 1.

TABLE 1

Rendezvous at	t	θ_C	θ_T	θ_L
0°	0	input	--	0°
	t_A	180°	180°	--
	t_B	--	0°	0°
180°	0	input	--	0°
	t_A	0°	0°	--
	t_B	--	180°	180°

As mentioned previously, if ΔV_{LO} is specified, the LEM orbit is determined. When $\Delta V_{LO} = 0$ the nominal circular orbit results. When $\Delta V_{LO} = 99.26$ fps, the aposelene of the resulting orbit equals R_C and the orbit is the nominal Hohmann ascent. When $\Delta V_{LO} = 190$ fps, $P_L = P_C$ the period of the CSM in its nominal circular orbit. When $\Delta V_{LO} = 350$ fps, the aposelene extends to an altitude of approximately 300 NM above the lunar surface.

(1) Rendezvous at 180°. From Figure 2a we see that

$$a_T = \frac{R_C + R_{LB}}{2} \tag{1}$$

$$P_T = \frac{2\pi}{\sqrt{\mu}} a_T^{3/2}; \quad P_C = \frac{2\pi}{\sqrt{\mu}} R_C^{3/2}; \quad P_L = \frac{2\pi}{\sqrt{\mu}} a_L^{3/2} \tag{2}$$

The figure also shows that $t_f = 1/2\ P_T$ plus the time for an integral number of revolutions, or

$$t_f = (J - 1/2)P_T, \quad J = 1, 2, 3,... \tag{3}$$

Similarly the rendezvous time (time from LEM cutoff to rescue) equals $1/2\ P_L$ plus the time for an integral number of revolutions, or

$$t_B = (I - 1/2)P_L, \quad I = 1, 2, 3,... \tag{4}$$

A restriction on the above is that the coast time $t_A = t_B - t_f$ must be greater than t_d, the time required for the real-time determination of the LEM orbit after LEM powered-flight cutoff. The value of t_A can be computed from

$$t_A = \left[\frac{\theta_{CA} - \theta_{CO}}{360} + K - 1 \right] P_C \tag{5}$$

since $\theta_{CA} = 0$,

$$t_A = \left(K - 1 - \frac{\theta_{CO}}{360} \right) P_C \tag{6}$$

Then

$$(I - 1/2)\, P_L = (J - 1/2)\, P_T + \left(K - 1 - \frac{\theta_{CO}}{360} \right) P_C \tag{7}$$

K is an integer which is dependent upon I and J. The formula can be determined by making equation (7) valid for the trivial case $P_L = P_T = P_C$ and $\theta_{CO} = 0$. Then

$$K = I - J + 1 \tag{8}$$

It is convenient from the computational point of view to replace the inequality $t_A > t_d$ by the inequality

$$K \geq 1 \text{ or } I \geq J \tag{9}$$

t_A can be monitored during any given calculation. It is thus unnecessary to decide upon a specific value for t_d at this time.

In equation (7), everything is known except θ_{CO}, I and J. A value of θ_{CO} can be calculated when ΔV_{LO}, I, and J are specified. Corresponding values of t_A, t_B, and ΔV_{CT} can also be obtained. These are plotted in Figures 3-7. Points on the curves at the same value of ΔV_{LO} represent identical geometrical situations, with only the number of revolutions in orbit changing. Therefore, ΔV_{CT} is constant for these different cases, and is so indicated on the figures.

The figures show that all the curves for which J is constant pass through a single point at $\Delta V_{LO} = 189.96$, which is the synchronous case mentioned earlier. This can be deduced from the above equations as follows:

Let $P_C = P_L$.

Then

$$(I - 1/2)\ P_C = \left(K-1 - \frac{\theta_{CO}}{360}\right) P_C + (J - 1/2)\ P_T$$

$$\theta_{CO} = 360\ (J - 1/2)\ R$$

where $R = \frac{P_T}{P_C} - 1 = \left(\frac{R_C + R_{LB}}{2R_C}\right)^{3/2} - 1 = .0532$

since R_{LB} = 6,625,600 ft. and R_C = 6,189,500 ft.

The fact that θ_{CO} increases with increasing J can be explained physically by noticing that as J is increased by 1 the number of revolutions in the transfer orbit is increased by one while the number of revolutions in the CSM orbit is reduced by one. Since the period of the transfer orbit is larger than the period of the CSM, a larger value for t_B would result unless θ_{CO} is increased by a compensating amount.

The total time t_B increases somewhat along a curve as ΔV_{LO} increases. Only the values at ΔV_{LO} = -53 fps and ΔV_{LO} = 350 fps are indicated on the curves, the variation along the curve being very nearly linear. The coast time t_A is also indicated. Note that t_A is negative for one of each family of curves. This should be interpreted as a case where the CSM begins its transfer maneuver before the LEM reaches the end of powered flight. Although this is not a practical maneuver, it is included for the sake of completeness.

(2) Rendezvous at 0°. From Figure 2b we see that

$$a_T = \frac{R_C + R_{LP}}{2}$$, (note that a_T is independent of ΔV_{LO})

P_T, P_C, and P_L are computed as before from equation (2)

$$t_f = (J - 1/2)\ P_T,\ J = 1, 2, 3,\ldots \text{ as before}$$

$$t_B = IP_L,\quad I = 1, 2, 3,\ldots \tag{10}$$

$$t_A = \left[\frac{\theta_{CA} - \theta_{CO}}{360} + K\text{-}1 \right] P_C \text{ as before}$$

since $\theta_{CA} = 180°$,

$$t_A = \left(K - 1/2 - \frac{\theta_{CO}}{360} \right) P_C,\ K = I - J + 1 \tag{11}$$

$$IP_L = \left(K - 1/2 - \frac{\theta_{CO}}{360} \right) P_C + (J - 1/2)\ P_T \tag{12}$$

As before, everything is known in equation (12) except θ_{CO}, I and J. Values of ΔV_{CT}, t_B and t_A are plotted in Figures 8-12 using ΔV_{LO} and θ_{CO} as coordinate variables. The remarks about Figures 3-7 also apply to Figures 8-12. The value of θ_{CO} for the synchronous case is given by the same equation as before, i.e.,

$$\theta_{CO} = 360\ (J - 1/2)\ R$$

$$\text{with } R = \left(\frac{R_C + R_{LP}}{2R_C} \right)^{3/2} - 1 = -.0522$$

Since $R_{LP} = 5{,}753{,}400$ ft.

Note that θ_{CO} is now negative, and becomes more negative as J increases.

Each curve of Figures 3-12 shows a relation between ΔV_{LO} and θ_{CO} which characterizes a collinear transfer orbit. Values for ΔV_{CT}, t_A, and t_B are also indicated. The relation between ΔV_{CT} and ΔV_{LO} is summarized in Figure 13. If a point is specified which does not lie on one of the curves it is impossible to find a corresponding collinear transfer orbit. In the original formulation of the problem in Section C, ΔV_{LO} and θ_{CO} were to be independent variables. However, the additional assumption that LEM and transfer orbits are collinear results in the indicated relationship between these two variables. The fact that this relation is multi-valued comes from cyclic nature of the rendezvous process.

Figure 14 is a superposition of all curves of Figures 3-12 for which t_B is less than 5 hours, and Figure 15 displays the same information with t_B less than 10 hours. In all cases t_A was greater than approximately 1-1/2 hours. Each curve in Figures 13 and 14 represents a discrete situation, the space between curves being made up of non-collinear cases. In an actual abort situation it may not be necessary to regard both θ_{CO} and ΔV_{LO} as entirely independent but instead one or the other or both could be varied somewhat so that a collinear rescue orbit is always used. A

comparatively low ΔV_{CT} will result. If this is not possible either a three-burn or a non-collinear maneuver could be used. (The latter is discussed in the next section.) In any case Figures 14 and 15 can be used to establish approximate bounds on θ_{CO} for any ΔV_{LO}. These will be pessimistic bounds since a third burn or the use of non-collinear maneuvers will extend the usable area in addition to making available the space between the curves.

Referring to Figure 15 it can be seen that for a rescue in 10 hours or less and ΔV_{LO} = -50 fps, the range in θ_{CO} for possible rescue extends approximately from 30 to 240 degrees. ΔV_{CT} equals 250 fps for the collinear cases. When ΔV_{LO} = 350 fps the θ_{CO} range extends approximately from -220 to +30 degrees. ΔV_{CT} is less than 350 fps for the collinear cases. The synchronous case of ΔV_{LO} = 190 fps is undesirable because the convergence of the curves to a point here makes the spacing between curves greater and consequently it would be more difficult to achieve the collinear case by adjusting ΔV_{LO} or θ_{CO} slightly. The same characteristics are exhibited for ΔV_{LO} = 99 fps, which also has the least range in θ_{CO}. For a rescue in 5 hours or less the ranges given above are reduced by a factor of about 2-1/2. The curves are also much farther apart, an undesirable situation as explained above.

F. NON-COLLINEAR CASES

If θ_{CO}, ΔV_{LO}, t_A and t_B are given, the transfer orbit and also ΔV_{CT} can be determined. It is then possible to vary t_A and t_B until ΔV_{CT} is a minimum. This calculation is described in detail in Appendix II. A computer program using these equations was written and used as a tool for numerically exploring some of the non-collinear orbits which make up the spaces between the curves of Figures 3-12. (An analytic approach would be more satisfactory, but the equations are too complicated to permit this.)

Referring to Figure 8, the point identified by X with ΔV_{LO} = 50.8 fps and θ_{CO} = 18.7° was arbitrarily selected for the investigation. t_A equaled .91 hours and t_B equaled 1.88 hours as computed from equations (11) and (10). When these numbers were used as inputs to the computer program a transfer orbit was determined which had its periselene at 0°, the optimized values for t_A and t_B were very nearly equal to the original values read in, and ΔV_{CT} equaled the 146 fps indicated in Figure 8. These results indicate that as one might expect a collinear transfer is one that has a minimum ΔV_{CT} with respect to t_A and t_B.

θ_{CO} was now incremented by 1 degree (ΔV_{LO} remaining constant), and the optimum transfer orbit again determined, using the previous values of t_A and t_B as initial values. This incrementing of θ_{CO} and the calculation of the optimum transfer orbit was continued until point W of Figure 3 was reached. The results are shown in Figures 16 and 17.

As θ_{CO} is increased, all three angles of Figure 16 increase, which means that the transfer orbit, the transfer point and the rendezvous point all rotate until the 180° collinear case is reached at θ_{CO} = 37.5°. This point is indicated by Y in Figures 3 and 16. In the same way when θ_{CO} = 46.9° the transfer orbit has rotated 360°, and the 0° collinear case is obtained (point Z), and when θ_{CO} = 65.7° the 180° collinear case is reached (point W). For all these cases, with θ_{CO} varying from 12° to 66°, the minimized value for ΔV_{CT} remains remarkably constant at 146 fps. Figure 17 shows the variation of t_A and t_B with θ_{CO}. The important thing to note is that the values of t_B form a continuous curve connecting points X,Y,Z, and W, and there exists a θ_{CO} for any value of t_B between the extreme values.

Figures 18 and 19 give the results of the same computer program when ΔV_{LO} = 273 fps. The points U, R, and S correspond to similarly labeled points in Figure 3. Note that three separate families of curves are produced, each containing one of the above mentioned points. The value of ΔV_{CT} is also plotted in Figure 19 and as one might expect it has a minimum at the collinear cases. The behavior is quite different from the previous case where ΔV_{CT} was almost constant at the collinear value. To minimize ΔV_{CT} it is necessary to shift abruptly from one set of curves to another where two ΔV_{CT} curves cross, as at point P of the figure. This minimum

curve is drawn in Figure 19 as a solid curve, with the remainder shown dotted. At these points the corresponding value of t_B changes by the period of the LEM orbit. Note that the minimum ΔV_{CT} for this rescue strategy is less than 350 fps. However t_B is restricted to certain ranges of values, an undesirable condition.

The requirement that t_A and t_B be equal to their optimum values is more stringent than necessary since all that is required is that their values be close enough to the optimum so that ΔV_{CT} is less than the budget value. To explore some of these cases, the computer program was modified so that t_B was specified as a linear interpolation between two adjacent collinear points. One advantage of this scheme is that t_B is not restricted to certain ranges of values as was the case in the previous paragraph. Expressed mathematically, for non-collinear cases between U and S of Figure 3

$$t_B = t_U + \frac{t_S - t_U}{\theta_S - \theta_U} (\theta - \theta_U)$$

where t_U and θ_U are values of t_B and θ_{CO} at point U and t_S and θ_S are values of t_B and θ_{CO} at point S. t_A alone was varied until a minimum ΔV_{CT} was obtained. The program in this form is considerably faster than the previously described one because the optimization is one instead of two-dimensional. The results

are plotted in Figures 20 and 21 for ΔV_{LO} = 273 fps. All curves are now continuous, and the curve for t_B forms a straight line connecting point U and S, and S and R. This is simply a reflection of the input assumptions. However, ΔV_{CT} rises to a maximum of over 600 fps. It is interesting to note in Figures 18 and 19 that the U, R, and S families are identical in shape and for Figures 20 and 21 the portion of the curves between U and S is identical in shape to that between S and R.

The results given in this section should be considered only preliminary, but it seems reasonable to assume that for ΔV_{LO} less that 190 fps, non-collinear orbits with ΔV_{CT} < 680 fps can always be found between adjacent collinear orbits. When ΔV_{CT} lies between 190 and 350 fps a conservative policy would be to assume that non-collinear orbits are not necessarily available with ΔV_{CT} < 680 fps. As explained in Section E, the need for these non-collinear orbits can be eliminated by dropping the requirement that ΔV_{LO} and ΔV_{CT} be completely independent.

L P Gieseler

2013-LPG-mhk

L. P. Gieseler

Attachments
Appendices I and II
Figures 1-21

REFERENCES

1. Hsu Lo and L. P. Gieseler, "CSM Rescue of the LEM During Lunar Ascent", Bellcomm Memorandum for File, September 1, 1964.

APPENDIX I
List of Symbols

a_L, e_L, p_L	elements of LEM orbit
a_T, e_T, p_T	elements of transfer orbit
P_L, P_T, P_C	period of LEM, transfer, and CSM orbits, respectively, seconds
t_A	time required for the CSM to travel from point 0 to point A
t_B	time from LEM cutoff to rescue
t_f	$t_B - t_A$

Note: Time is in seconds for the equations. It has been converted to hours for the figures.

R_C	radius of CSM orbit
R_{LA}, R_{LB}	radial distance of LEM at t_A and t_B respective
R_{TA}, R_{TB}	radial distance of transfer vehicle at t_A and t_B respectively
R_{LP}	radial distance to LEM orbit perisélene
R_{LQ}	radial distance to LEM orbit aposelene
V_{std}	circular velocity of the standard LEM parking orbit (alt. = 50,000 ft)
ΔV_{LO}	LEM velocity excess over V_{std} when t = (
ΔV_{CT}	total velocity increment required by t! for rescue
ΔV_{AR}, $\Delta V_{A\theta}$, ΔV_A	radial, tangential, and total velocit of CSM at point A

ΔV_{BR}, $\Delta V_{B\theta}$, ΔV_B	radial, tangential, and total velocity increment of CSM at point B
θ_{CO}, θ_{CA}, θ_{CB}	angular position of CSM when t = 0, t_A and t_B respectively. θ_{CO} is also the angle that the CSM leads the LEM when t = 0.
θ_{TO}, θ_{TA}, θ_{TB}	angular position of transfer vehicle when t = 0, t_A, and t_B respectively
θ_{LO}, θ_{LA}, θ_{LB}	angular position of LEM when t = 0, t_A, and t_B respectively
θ_{TP}	angular position of transfer orbit periselene
μ	gravitational constant of the moon, ft^3/sec^2 (1.7313997×10^{14} used)
ψ_A	true anomaly of point A on transfer orbit
α_s	sweep angle between points A and B

APPENDIX II

Equations Used in Program for Determining Non-Collinear Transfer Orbits

Input - μ, R_{LP}, R_C, ΔV_{LO}, θ_{CO}, and initial values of t_A and t_B

Output - Optimum value of ΔV_{CT} with respect to t_A and t_B, the corresponding values of t_A and t_B, and also θ_{CA}, θ_{LB}, R_{LB}, p_T, ψ_A, e_T, and θ_{TP}

Preliminary Calculations

$$V_{std} = \sqrt{\mu/R_{LP}}$$

$$V_{LO} = V_{std} + \Delta V_{LO}$$

$$a_L = (2\mu - V_{LO}^2 R_{LP})/\mu R_{LP}$$

$$e_L = 1 - R_{LP}/a_L$$

$$p_L = a_L(1 - e_L^2)$$

$$P_1 = 2\pi/\sqrt{\mu}$$

$$P_L = P_1 a_L^{3/2}$$

$$P_C = P_1 R_C^{3/2}$$

Transfer Orbit Calculations

$$t_f = t_B - t_A$$

$$\theta_{CA} = 360\ t_A/P_C$$

$$\left.\begin{aligned} \theta_{LB} &= f_1(a_L, e_L, \mu, t_B) \\ R_{LB} &= f_2(a_L, e_L, \mu, t_B) \end{aligned}\right\} \quad \text{subroutine SUB 1}$$

$$\alpha_s = \theta_{LB} - \theta_{CA}$$

$$\left.\begin{aligned} p_T &= g_1(\mu, R_C, R_{LB}, \alpha_s, t_f) \\ \psi_A &= g_2(\mu, R_C, R_{LB}, \alpha_s, t_f) \\ e_T &= g_3(\mu, R_C, R_{LB}, \alpha_s, t_f) \end{aligned}\right\} \quad \text{subroutine LAMBRT}$$

Determination of ΔV_{CT}

$$\Delta V_{AR} = e_T \sin \psi_A \sqrt{\mu/p_T}$$

$$\Delta V_{A\theta} = \left(\sqrt{p_T/R_C} - 1.0\right) \sqrt{\mu/R_C}$$

$$\Delta V_A = \sqrt{\Delta V_{AR}^2 + \Delta V_{A\theta}^2}$$

$$\Delta V_{BR} = \sqrt{\mu} \left[\frac{e_T \sin (\psi_A + \alpha_s)}{\sqrt{p_T}} - \frac{e_L \sin \theta_{LB}}{\sqrt{p_L}} \right]$$

$$\Delta V_{B\theta} = \sqrt{\mu} \frac{\left(\sqrt{p_T} - \sqrt{p_L}\right)}{R_{LB}}$$

$$\Delta V_B = \sqrt{\Delta V_{BR}^2 + \Delta V_{B\theta}^2}$$

$$\Delta V_{CT} = \Delta V_A + \Delta V_B$$

Improvement of t_A and t_B

Five values of ΔV_{CT} are determined according to the following table:

t_A	t_{Ak}	t_{Ak} + 100 sec	t_{Ak} - 100	t_{Ak}	t_{Ak}
t_B	t_{Bk}	t_{Bk}	t_{Bk}	t_{Bk} + 100	t_{Bk} - 100
ΔV_{CT}	ΔV_{CT1}	ΔV_{CT2}	ΔV_{CT3}	ΔV_{CT4}	ΔV_{CT5}

then

$$\Delta t_A = \frac{50(\Delta V_{CT2} - \Delta V_{CT3})}{2\Delta V_{CT1} - \Delta V_{CT2} - \Delta V_{CT3}}$$

$$\Delta t_B = \frac{50(\Delta V_{CT4} - \Delta V_{CT5})}{2\Delta V_{CT1} - \Delta V_{CT2} - \Delta V_{CT3}}$$

$$t_{Ak+1} = t_{Ak} + \Delta t_A$$

$$t_{Bk+1} = t_{Bk} + \Delta t_B$$

$$\theta_{TP} = \theta_{CA} - \psi_A$$

Go to "Transfer Orbit Calculations"

The program is arranged so that the absolute value of Δt_A or Δt_B cannot exceed 250 seconds. The iteration terminates when $|\Delta t_A| + |\Delta t_B|$ becomes less than 50 seconds or when the number of iterations exceeds 50. (The number of iterations does not normally exceed 10.) The equations used for the subroutines are given in Reference 1, equation (31) used in SUB 1, and Appendix I used for LAMBRT.

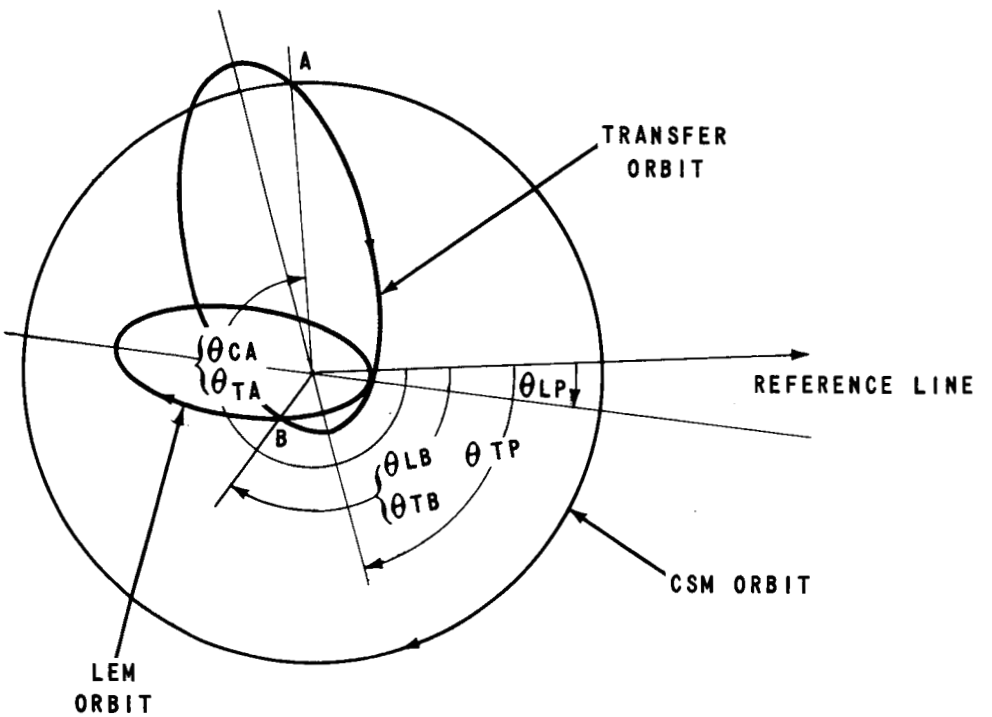

FIGURE I GENERAL RESCUE MANEUVER

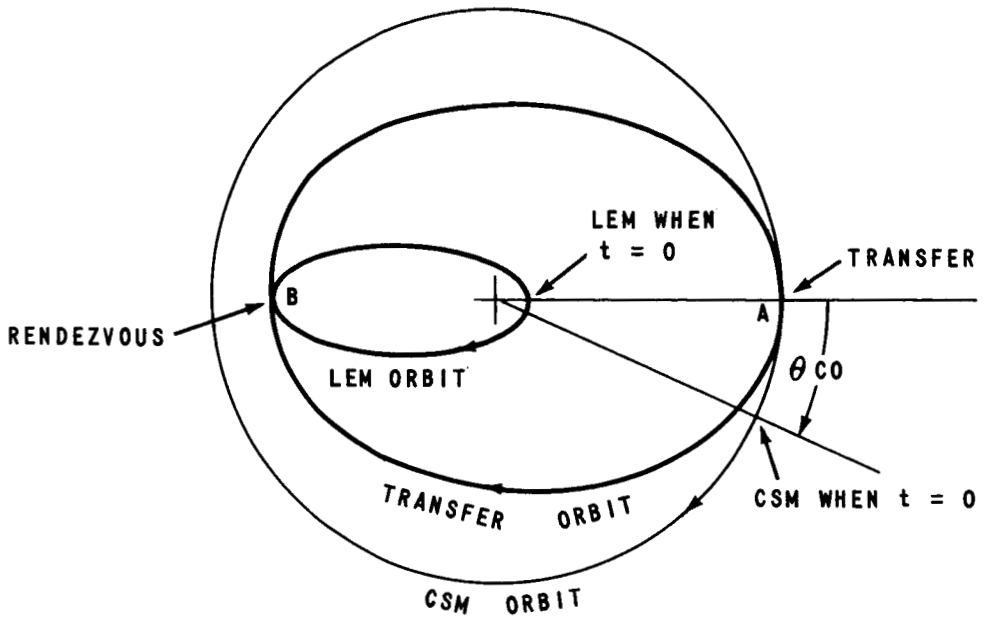

FIGURE 2a RENDEZVOUS 180° FROM LEM ORBIT PERISELENE

(COLLINEAR CASE)

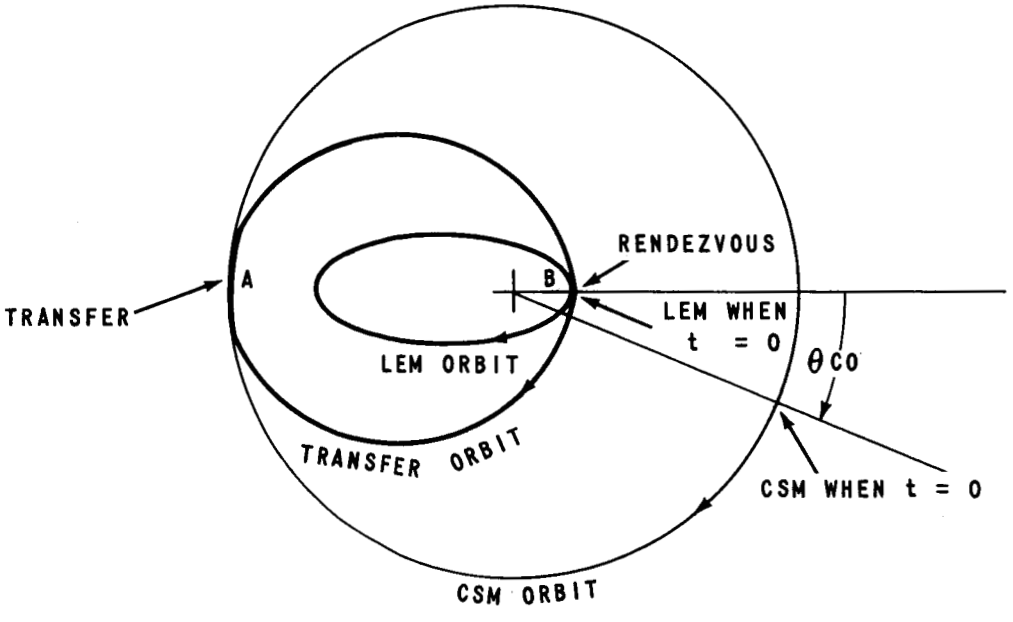

FIGURE 2b RENDEZVOUS AT LEM ORBIT PERISELENE (0°)

(COLLINEAR CASE)

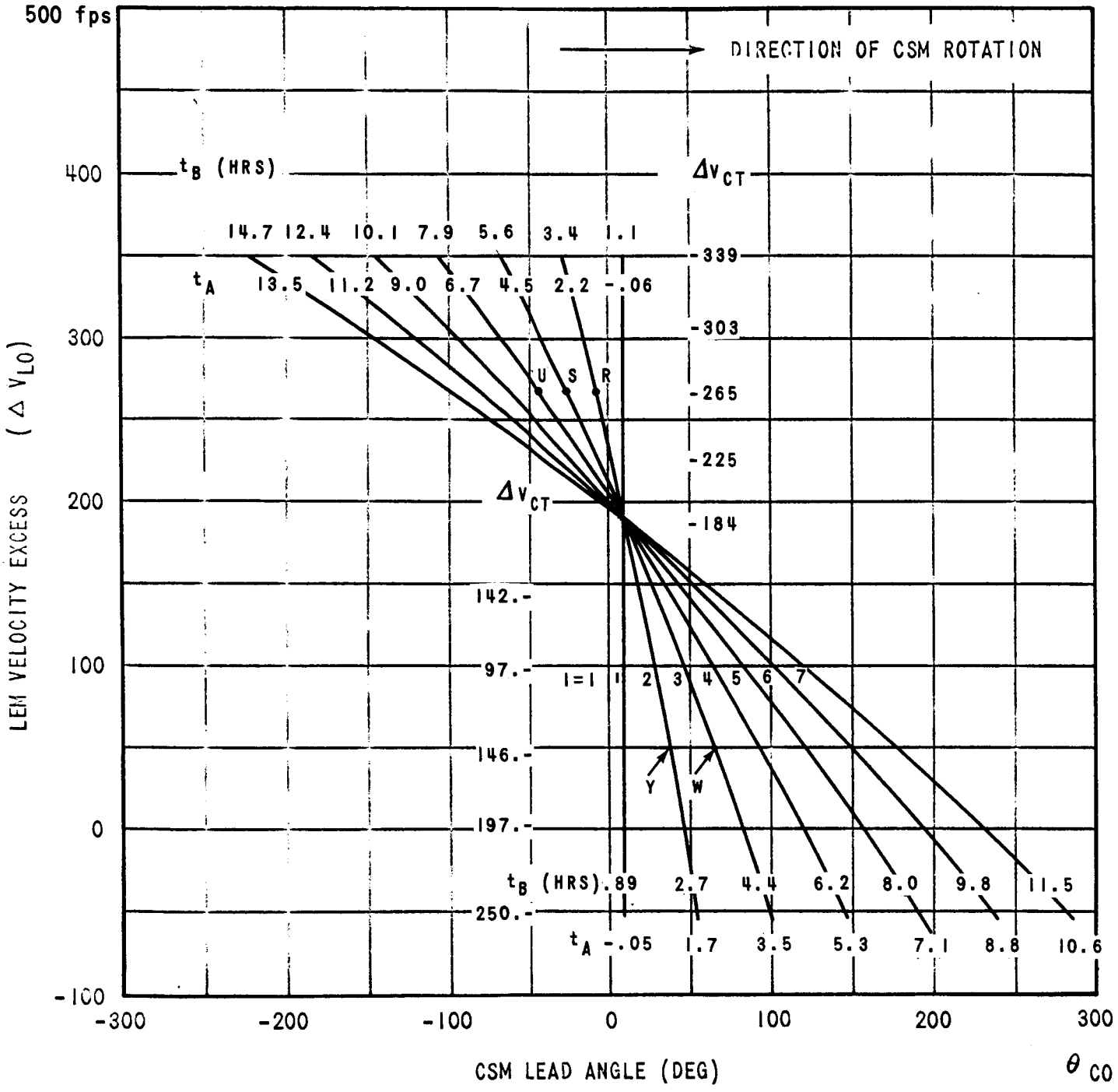

FIGURE 3 RENDEZVOUS AT 180 FROM LEM PERISELENE
(1/2 REVOLUTION IN TRANSFER ORBIT, I.E j = 1)

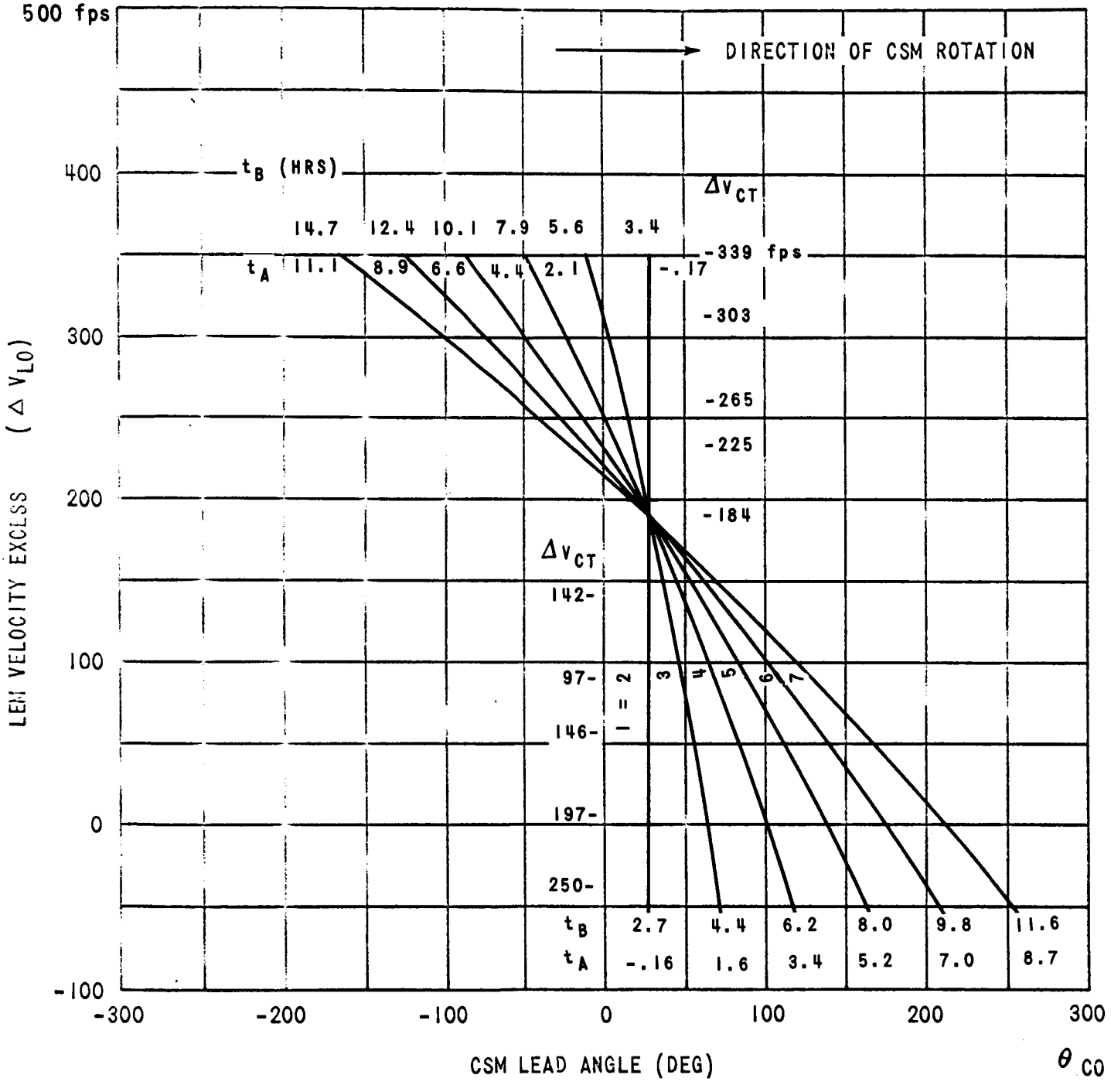

FIGURE 4 RENDEZVOUS AT 180° FROM LEM PERISELENE
(1½ REVOLUTION IN TRANSFER ORBIT, i.e., J = 2)

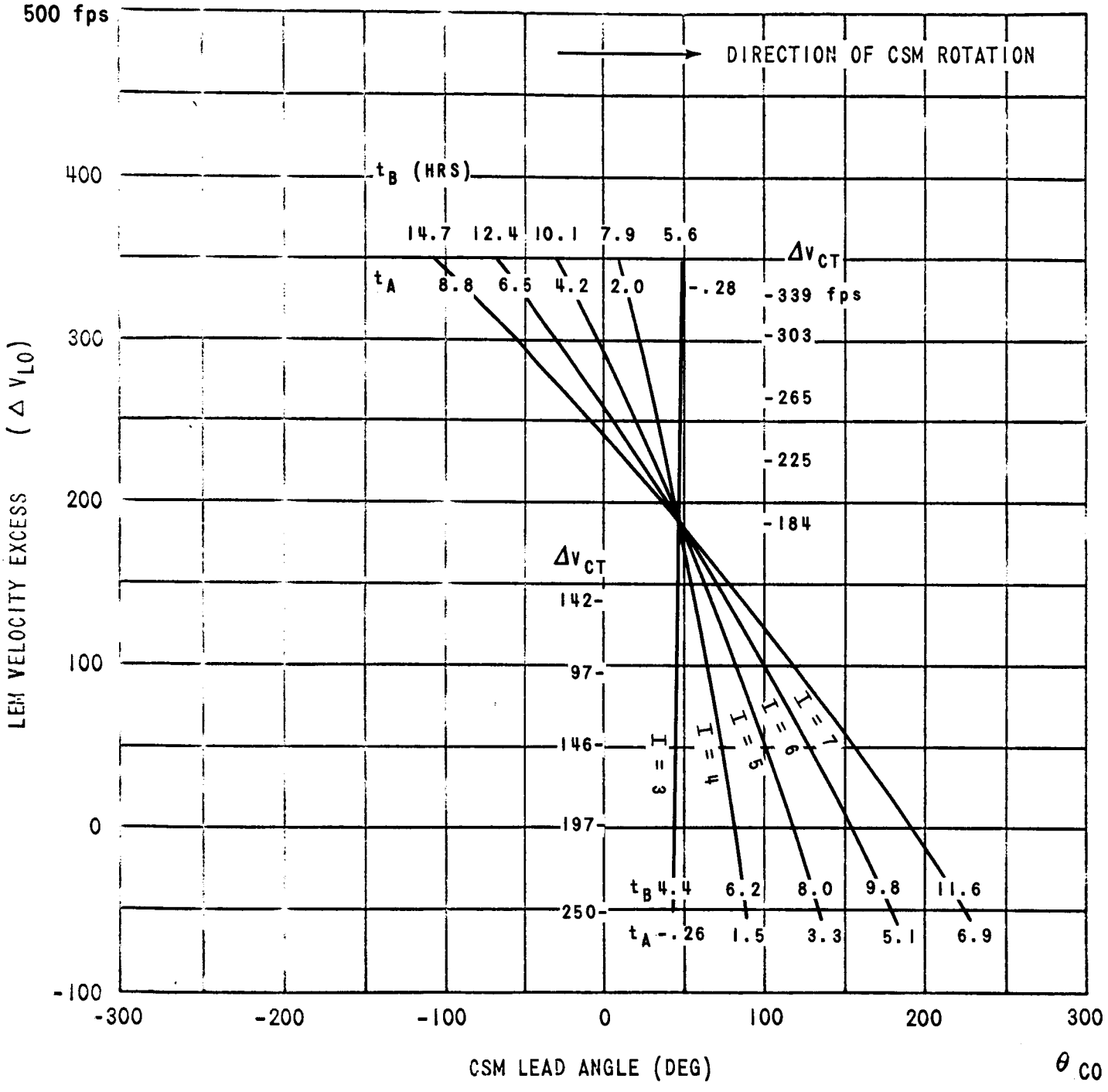

FIGURE 5 RENDEZVOUS AT 180 FROM LEM PERISELENE
(2-1/2 REVOLUTIONS IN TRANSFER ORBIT, I.E. j = 3)

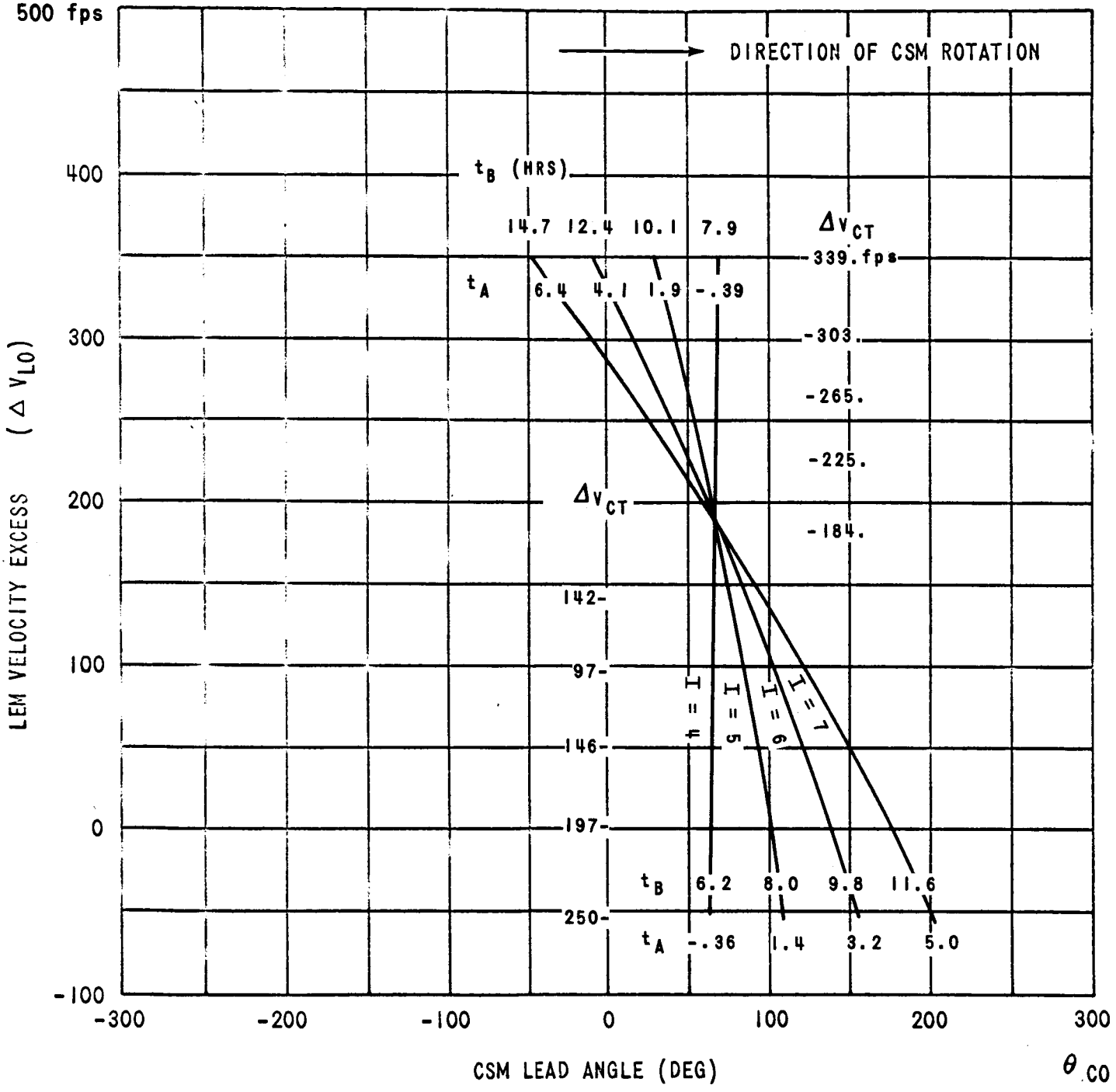

FIGURE 6 RENDEZVOUS AT 180° FROM LEM PERISELENE
(3-1/2 REVOLUTIONS IN TRANSFER ORBIT, I.E. j = 4)

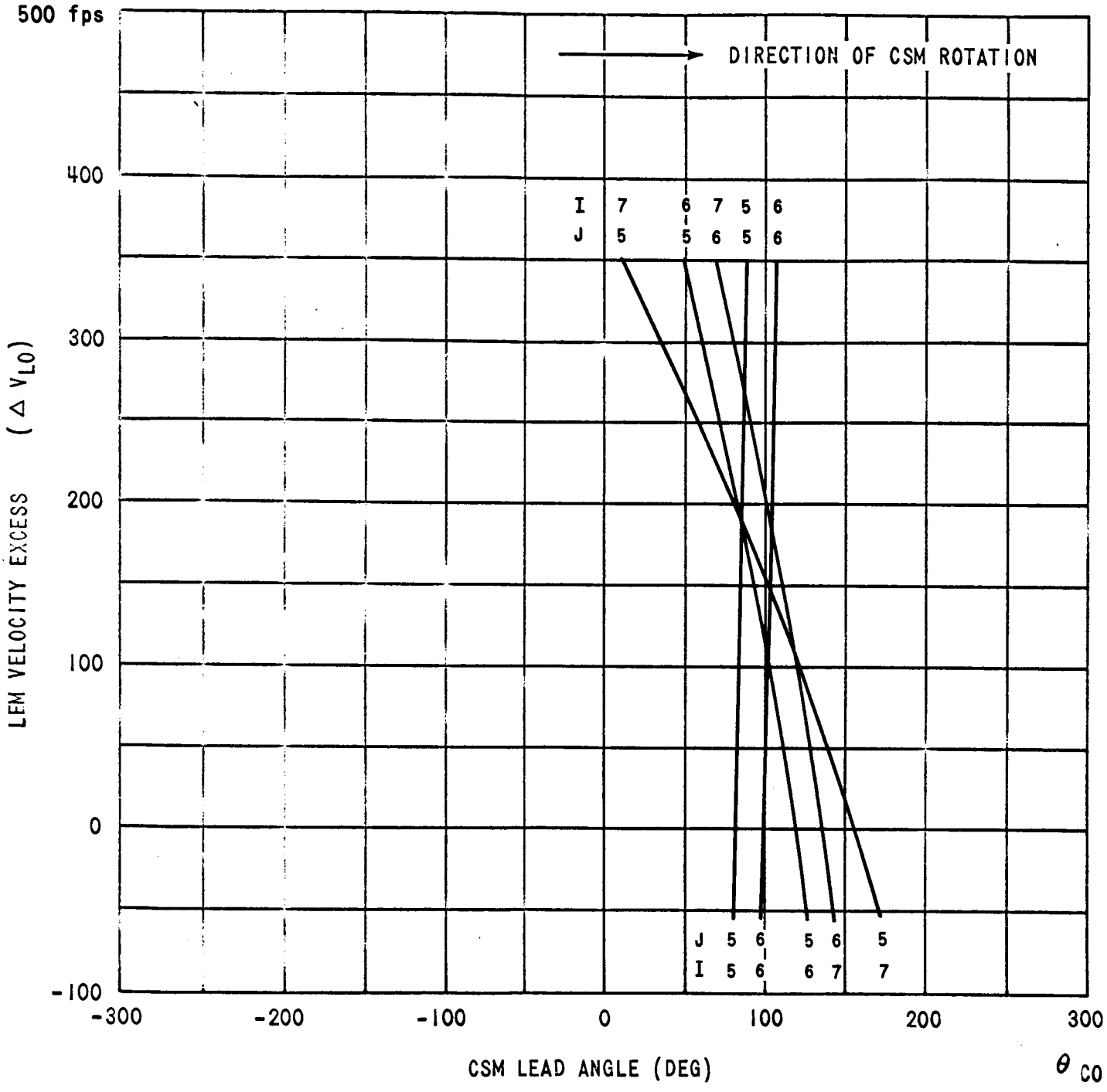

FIGURE 7 RENDEZVOUS AT 180° FROM LEM PERISELENE
(j = 5 AND 6)

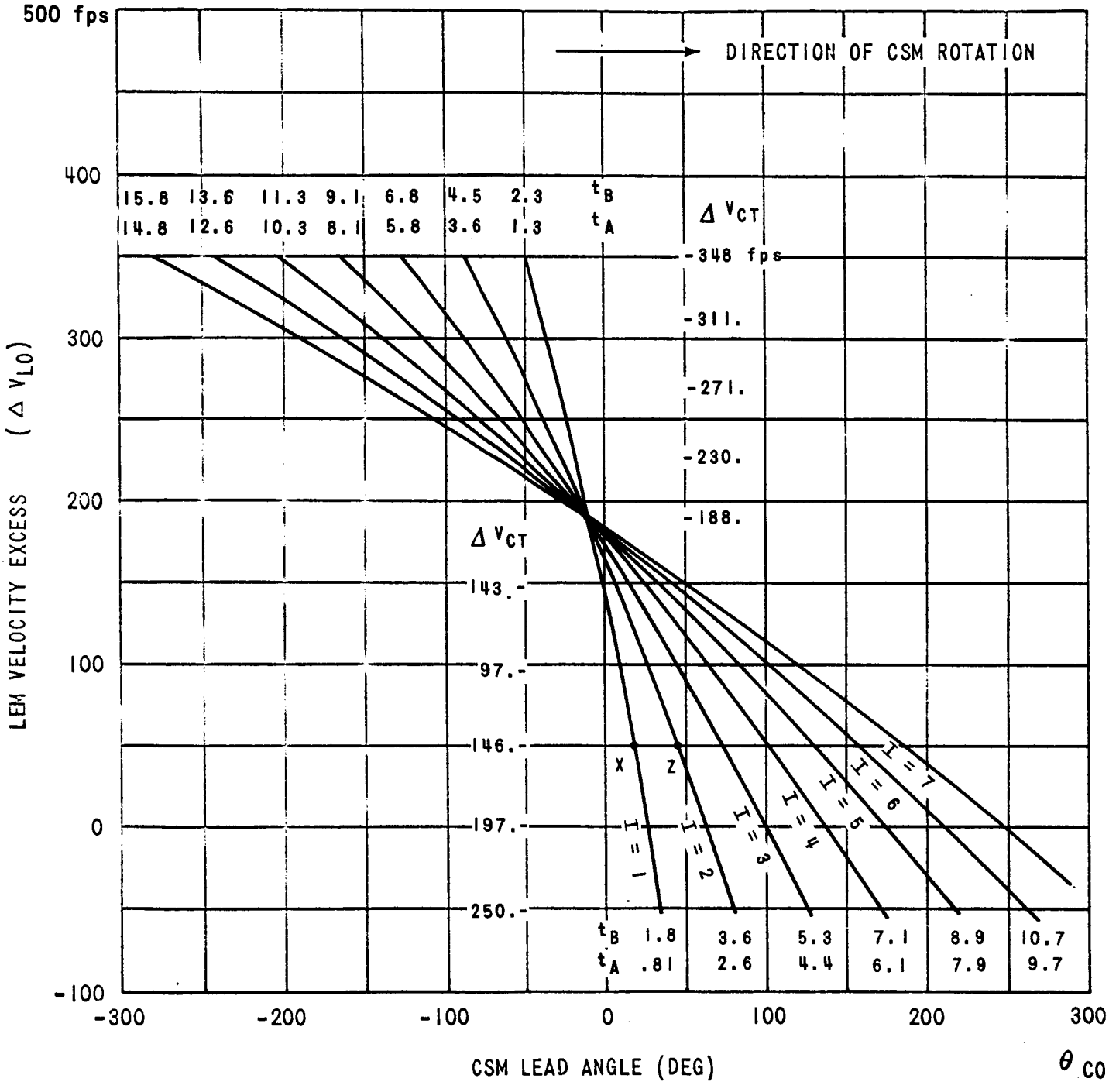

FIGURE 8 RENDEZVOUS AT 0^0
(1/2 REVOLUTION IN TRANSFER ORBIT, I.E. j = 1)

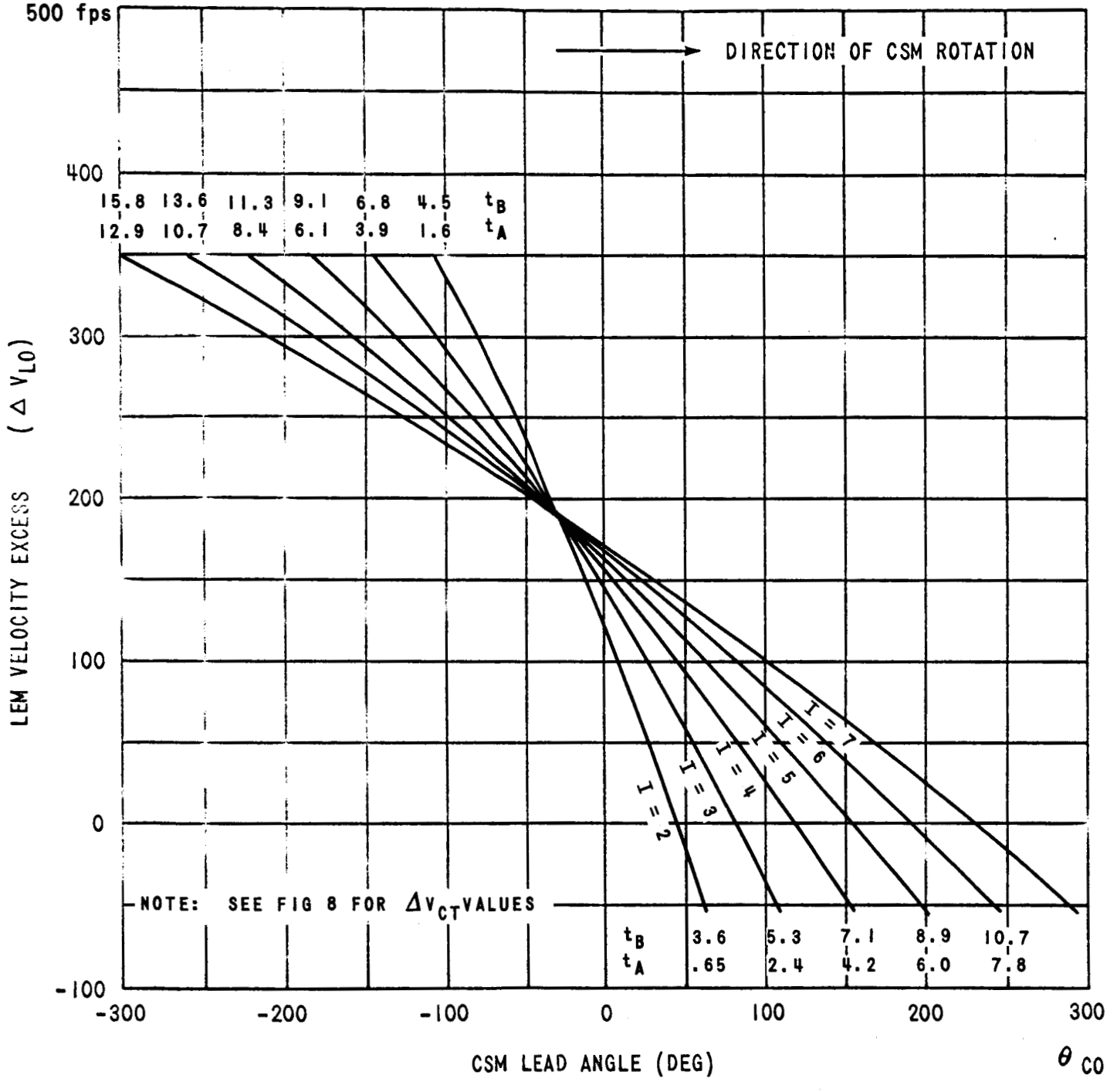

FIGURE 9 RENDEZVOUS AT 0^0

(1-1/2 REVOLUTIONS IN TRANSFER ORBIT I.E. j = 2)

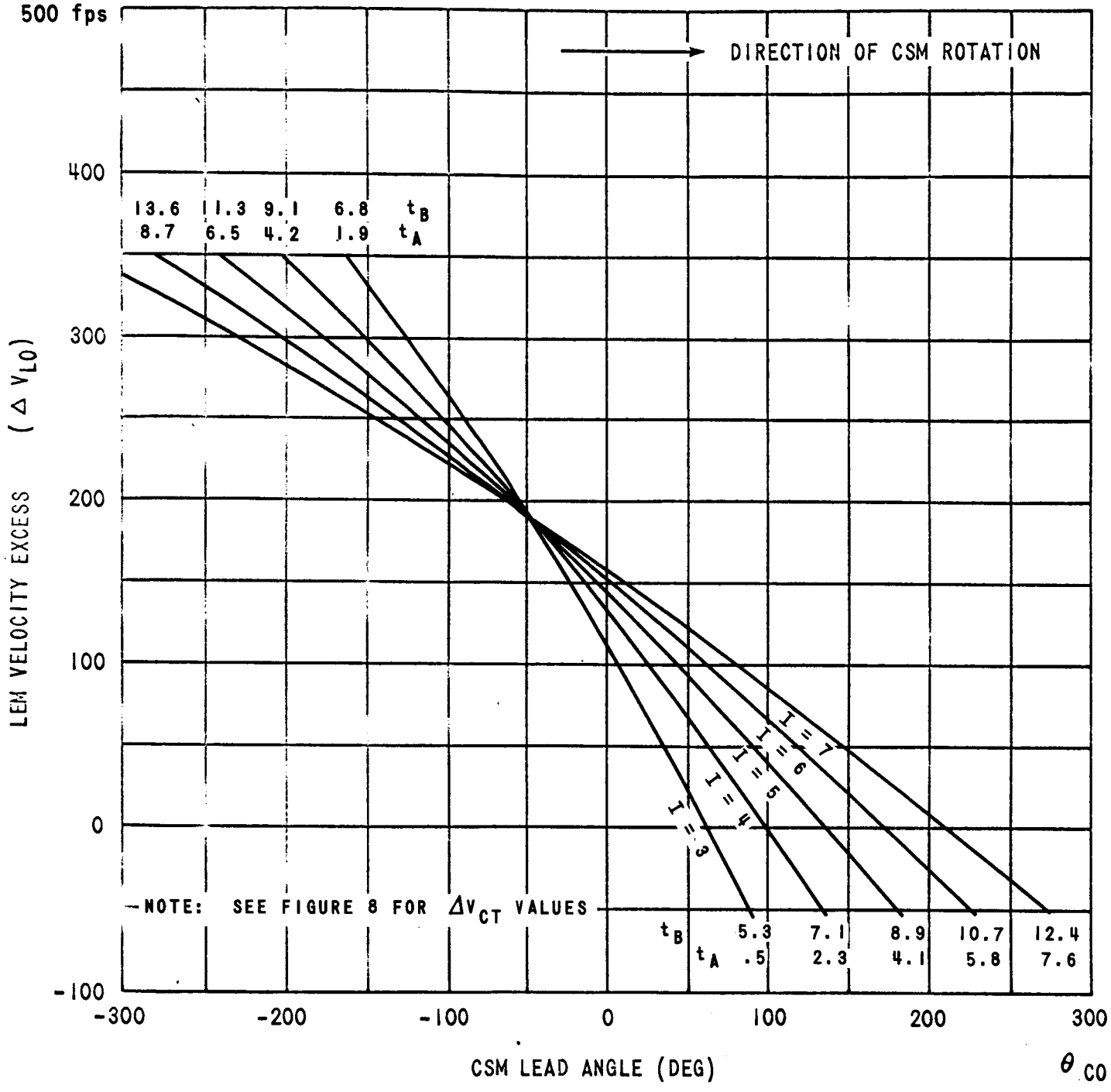

FIGURE 10 RENDEZVOUS AT 0^0
(2-1/2 REVOLUTIONS IN TRANSFER ORBIT, I.E. j = 3)

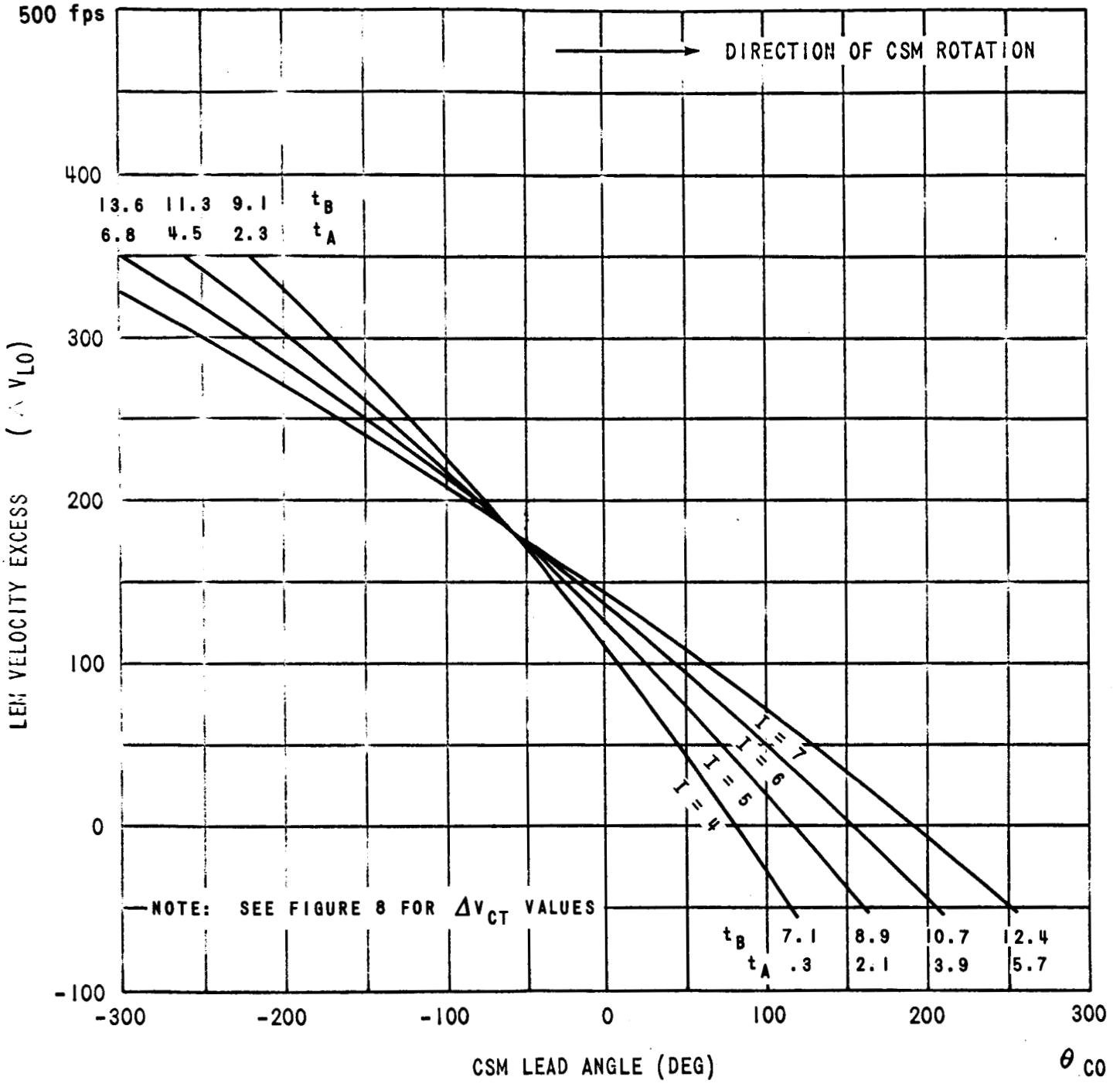

FIGURE 11 RENDEZVOUS AT 0°
(3-1/2 REVOLUTIONS IN TRANSFER ORBIT, I.E. j = 4)

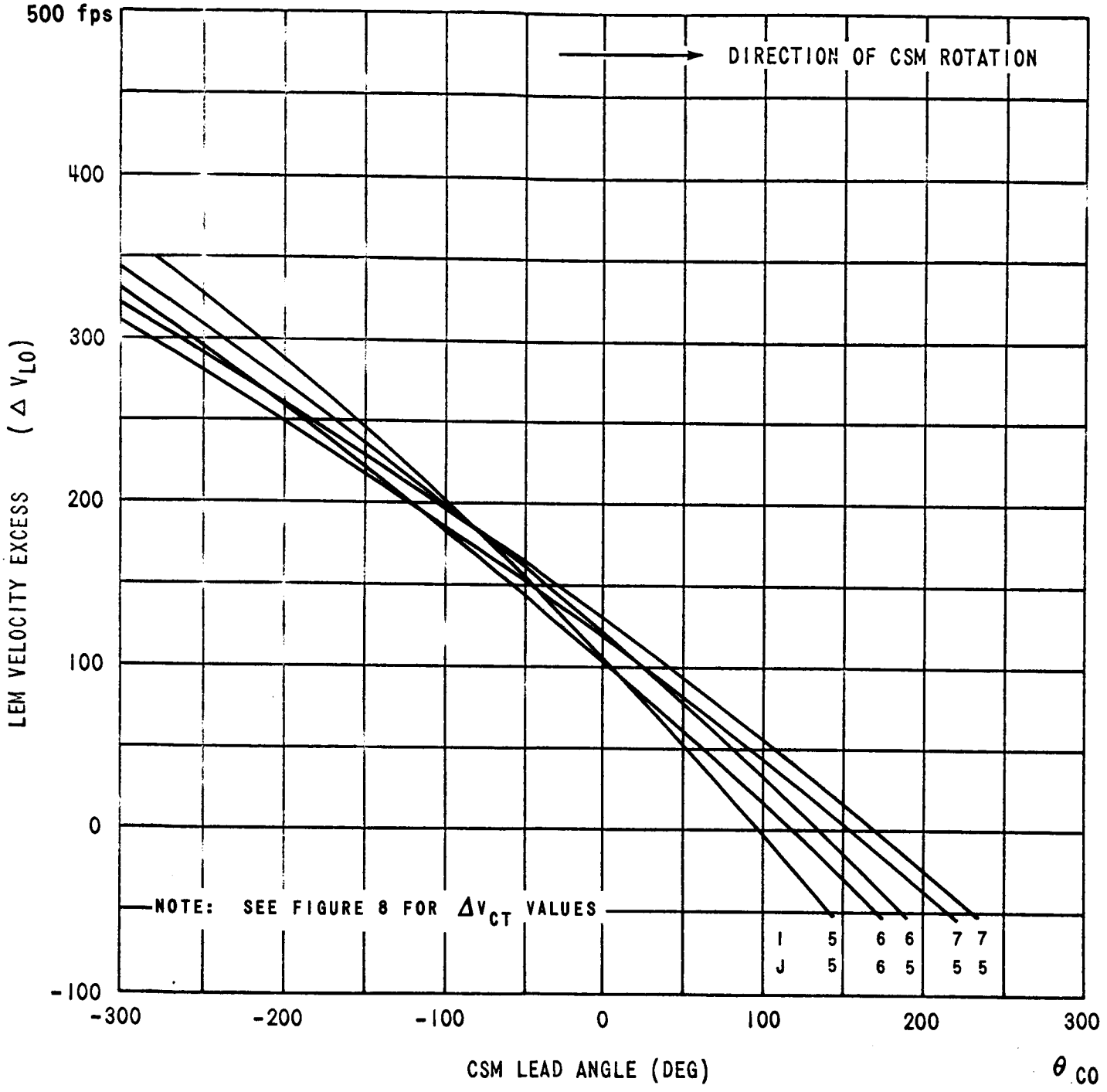

FIGURE 12 RENDEZVOUS AT 0°
(j = 5 AND 6)

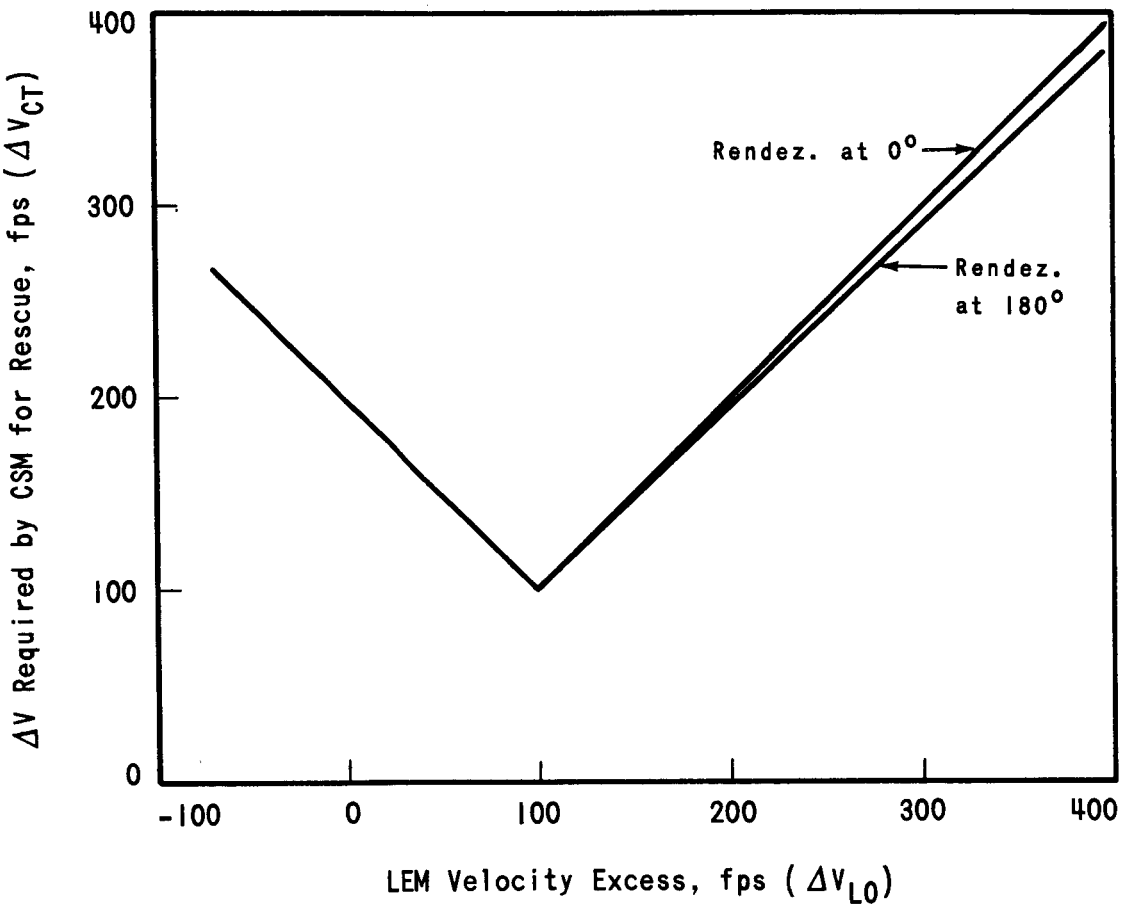

FIGURE 13 VARIATION OF CSM ΔV WITH LEM VELOCITY EXCESS

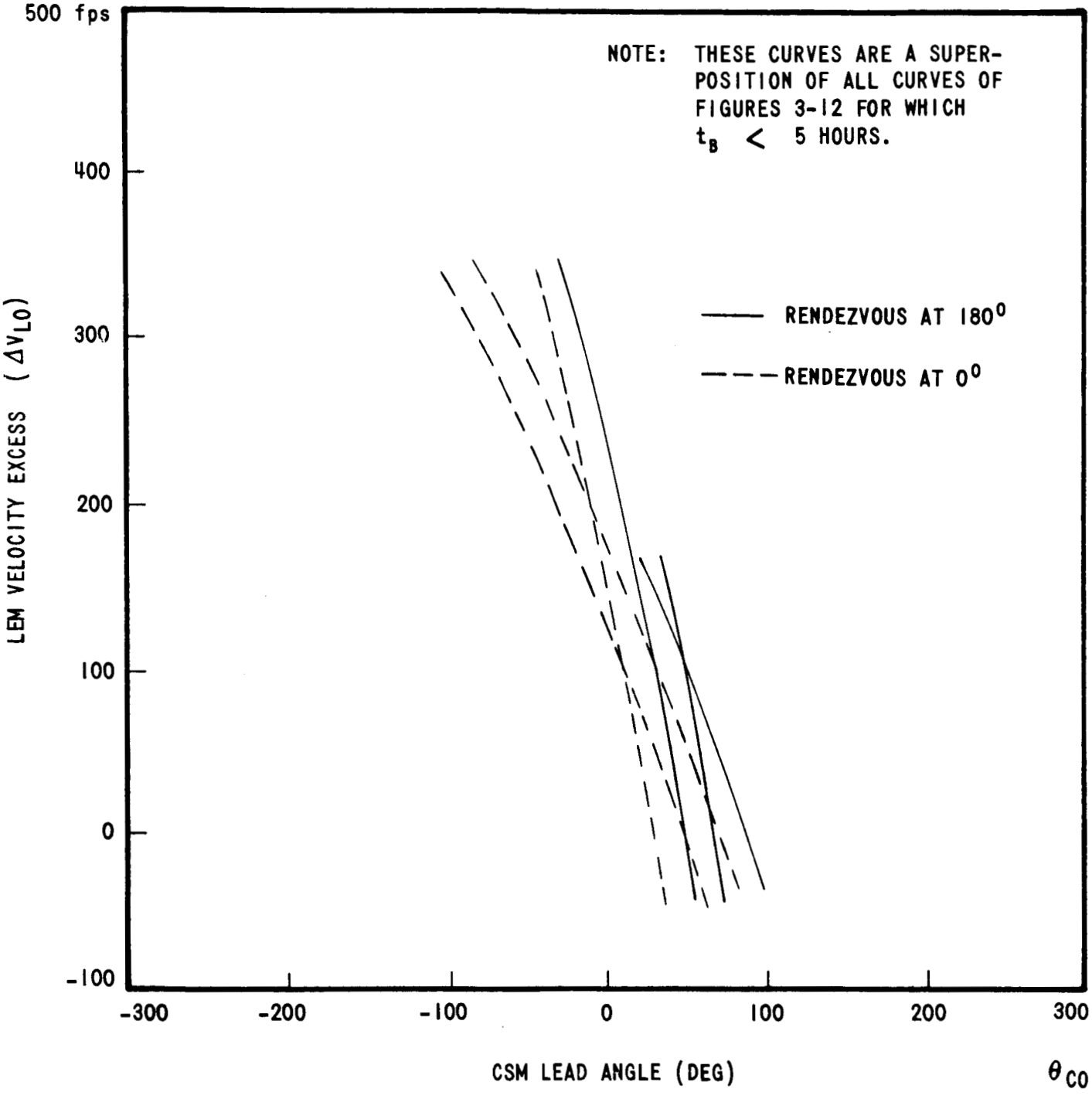

FIGURE 14 RESCUE IN FIVE HOURS OR LESS

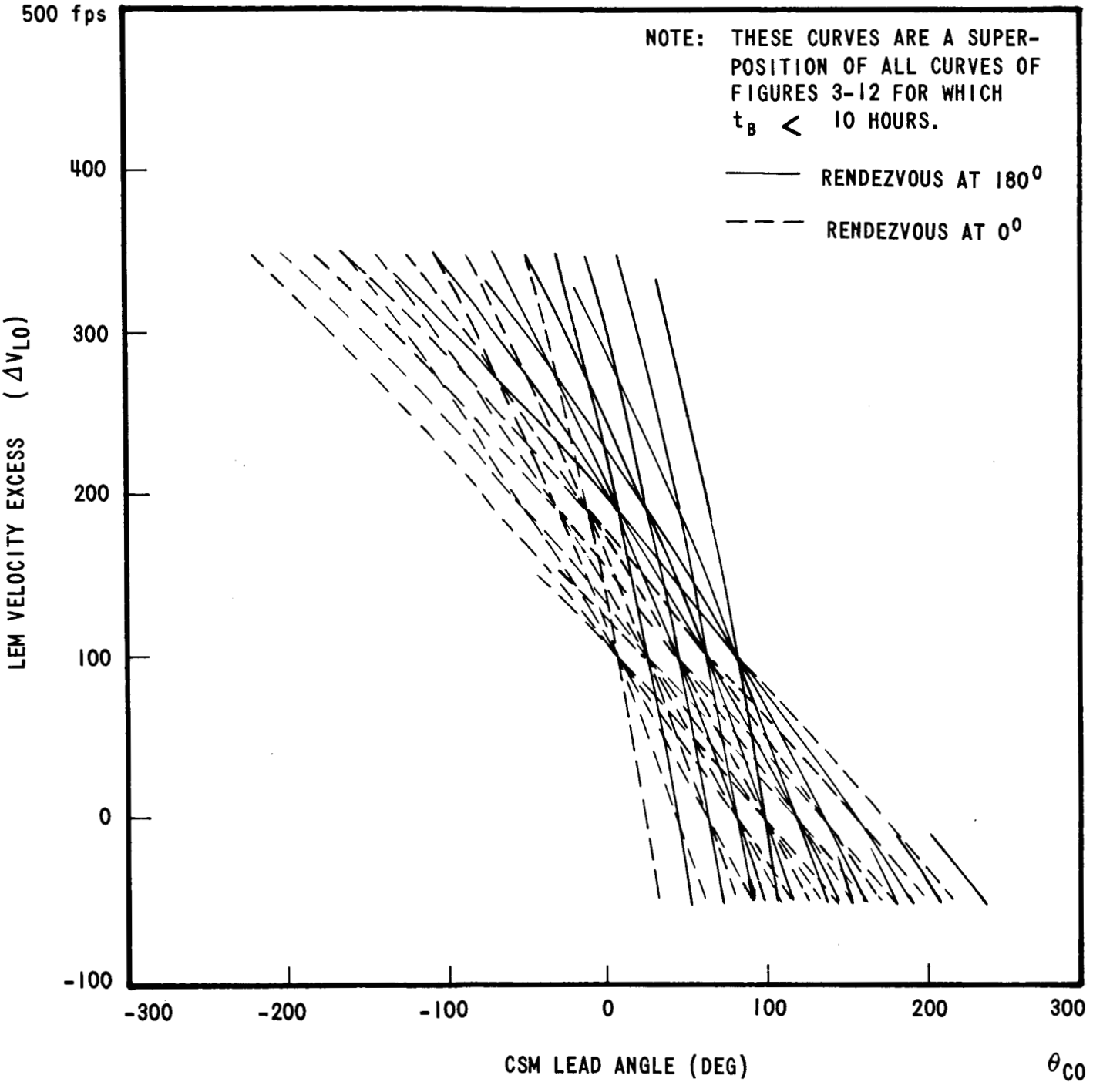

FIGURE 15 RESCUE IN 10 HOURS OR LESS

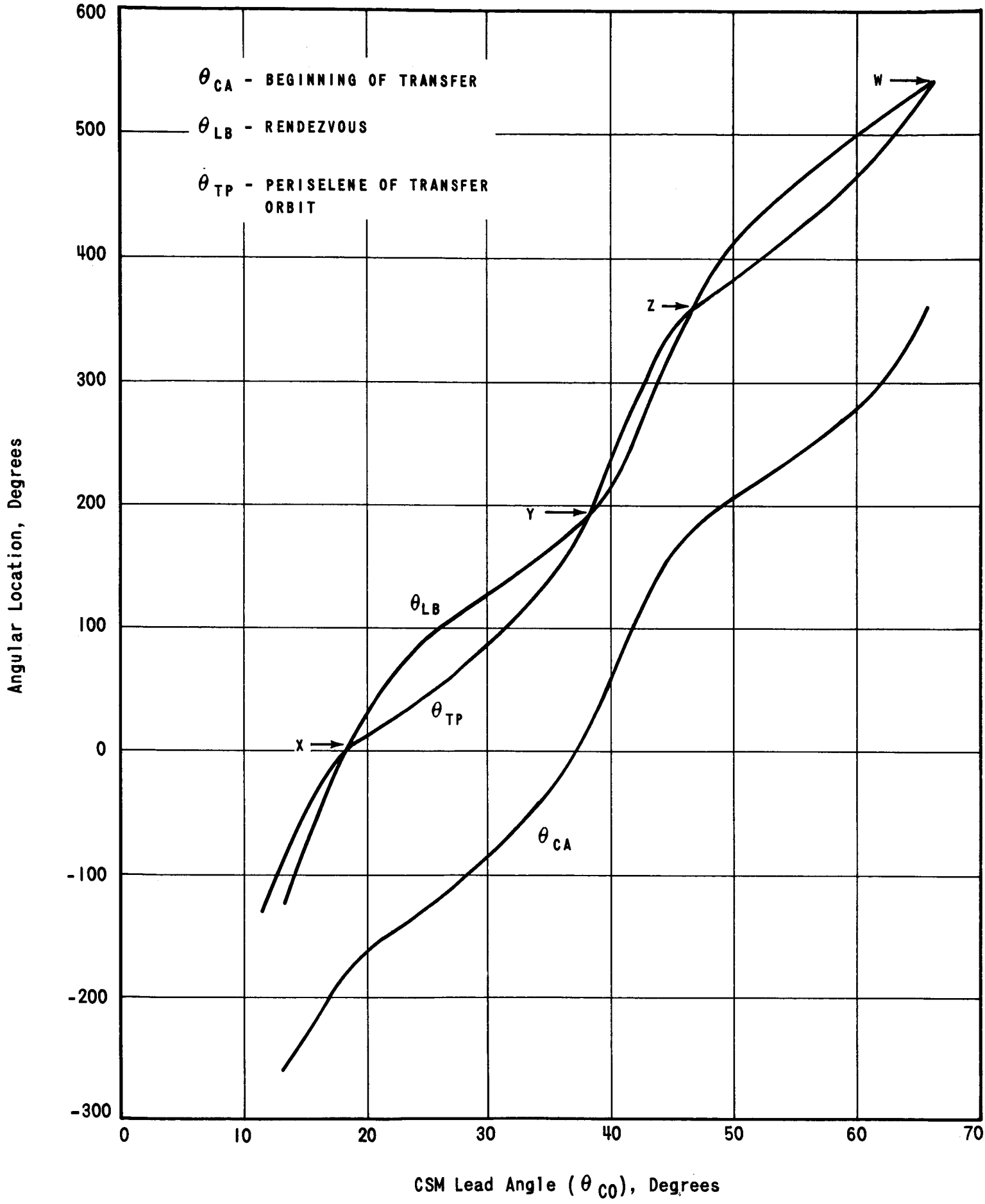

FIGURE 16 NON-COLLINEAR TRANSFER ORBITS, ΔV_{LO} = 50.8 fps
(ANGULAR LOCATIONS)

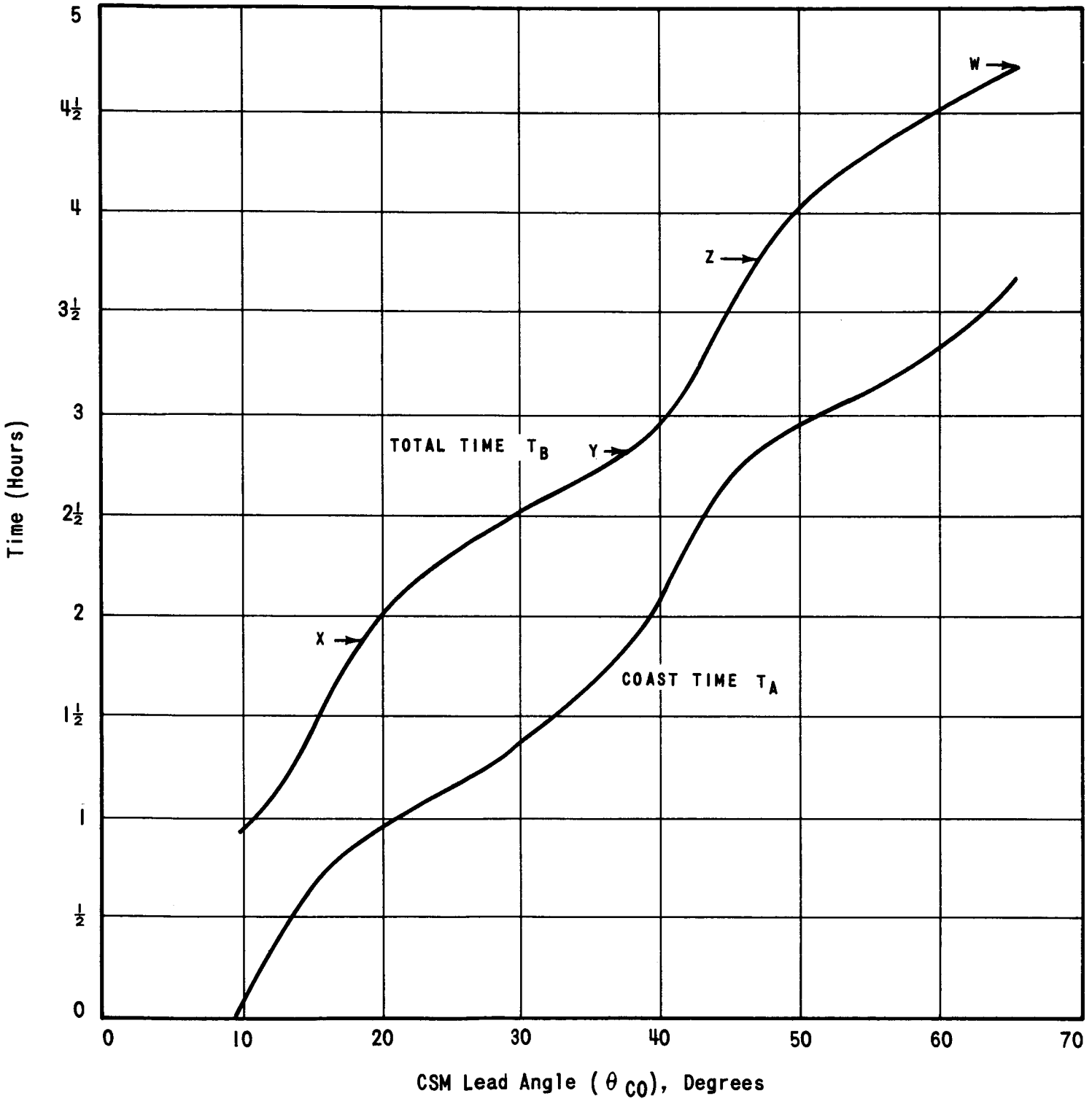

FIGURE 17 NON-COLLINEAR TRANSFER ORBITS, ΔV_{LO} = 50.8 fps
(COAST AND TOTAL TIMES)

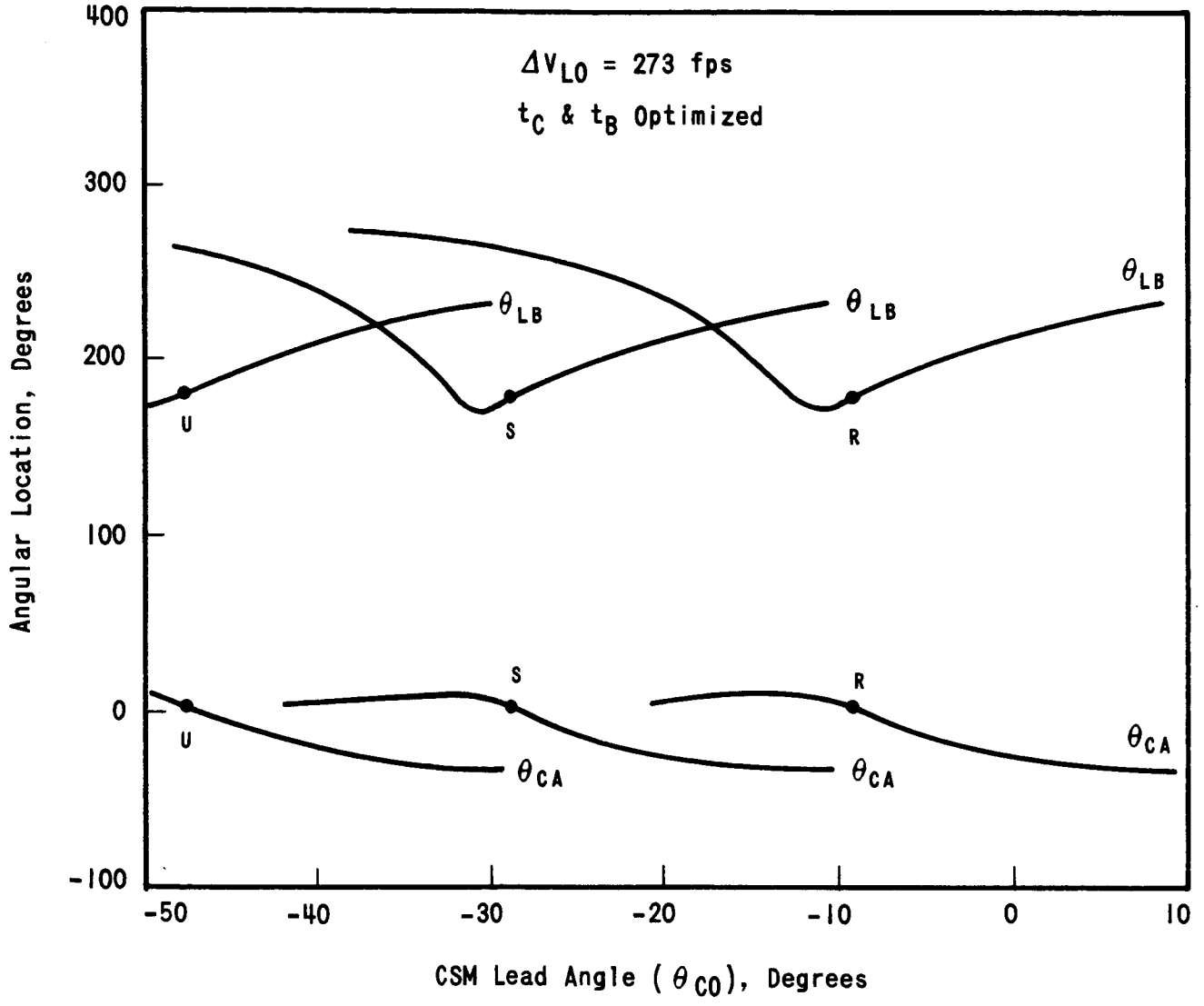

FIGURE 18 NON-COLLINEAR TRANSFER ORBIT, ΔV_{LO} = 273 fps
(ANGULAR LOCATIONS)

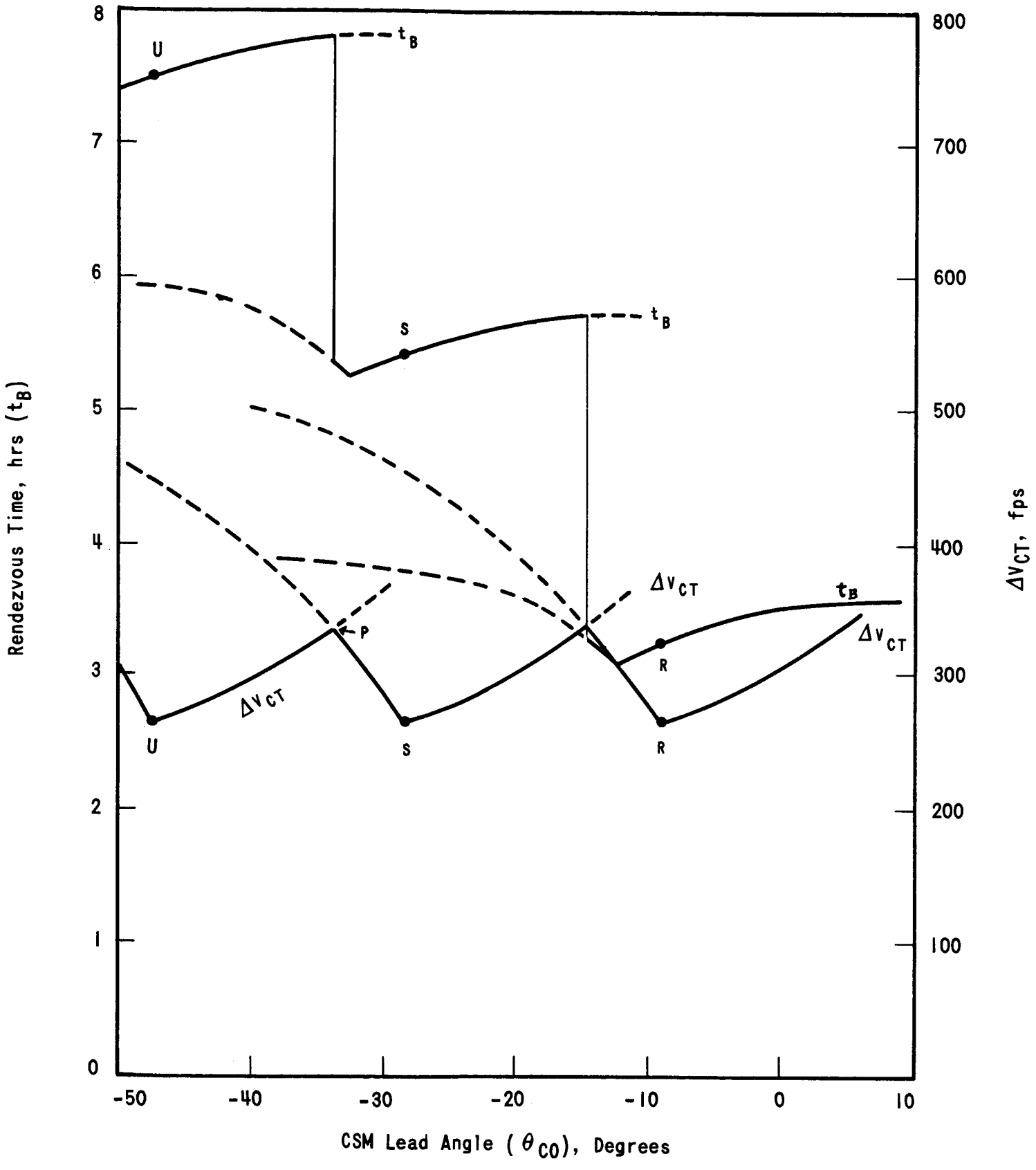

FIGURE 19 NON-COLLINEAR ORBITS, ΔV_{LO} = 273 fps

(t_B AND ΔV_{CT})

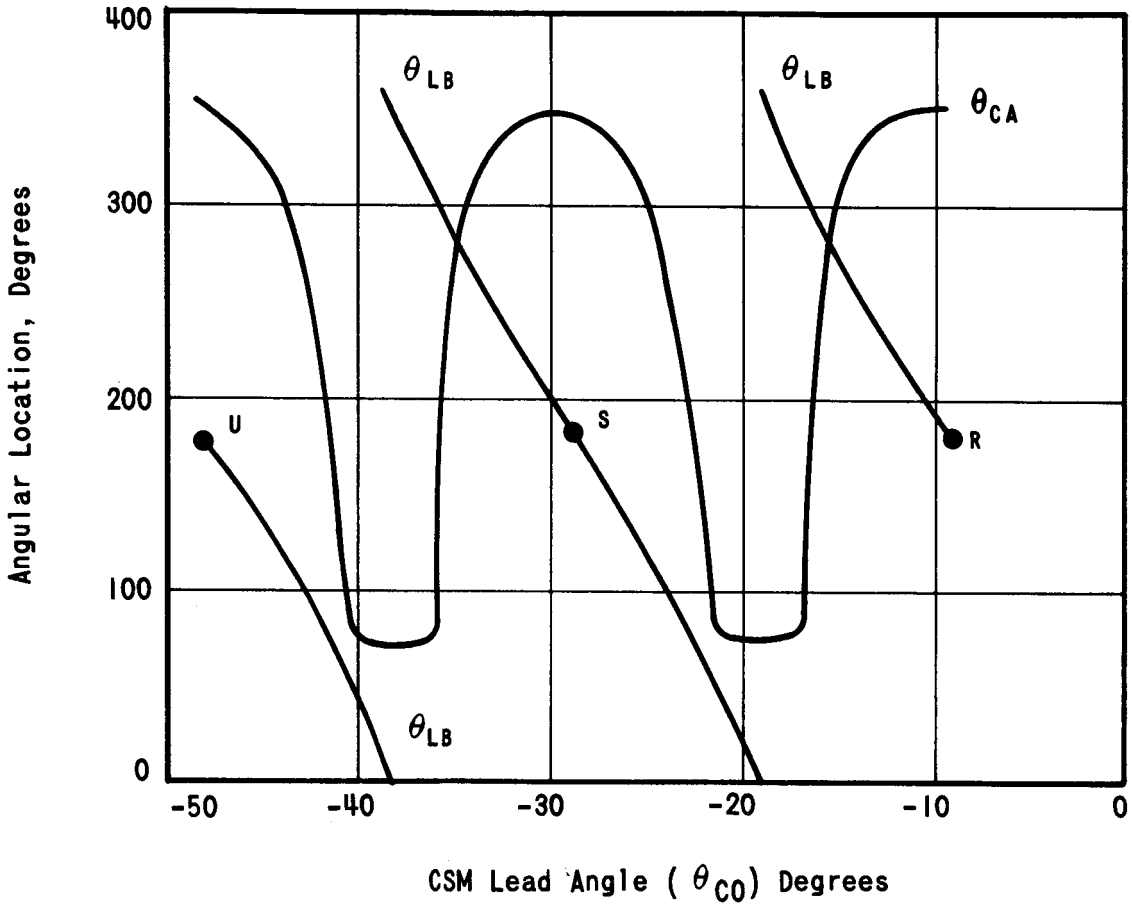

FIGURE 20 NON-COLLINEAR TRANSFER ORBITS, ΔV_{LO} = 273 fps

(T_B NOT OPTIMIZED)

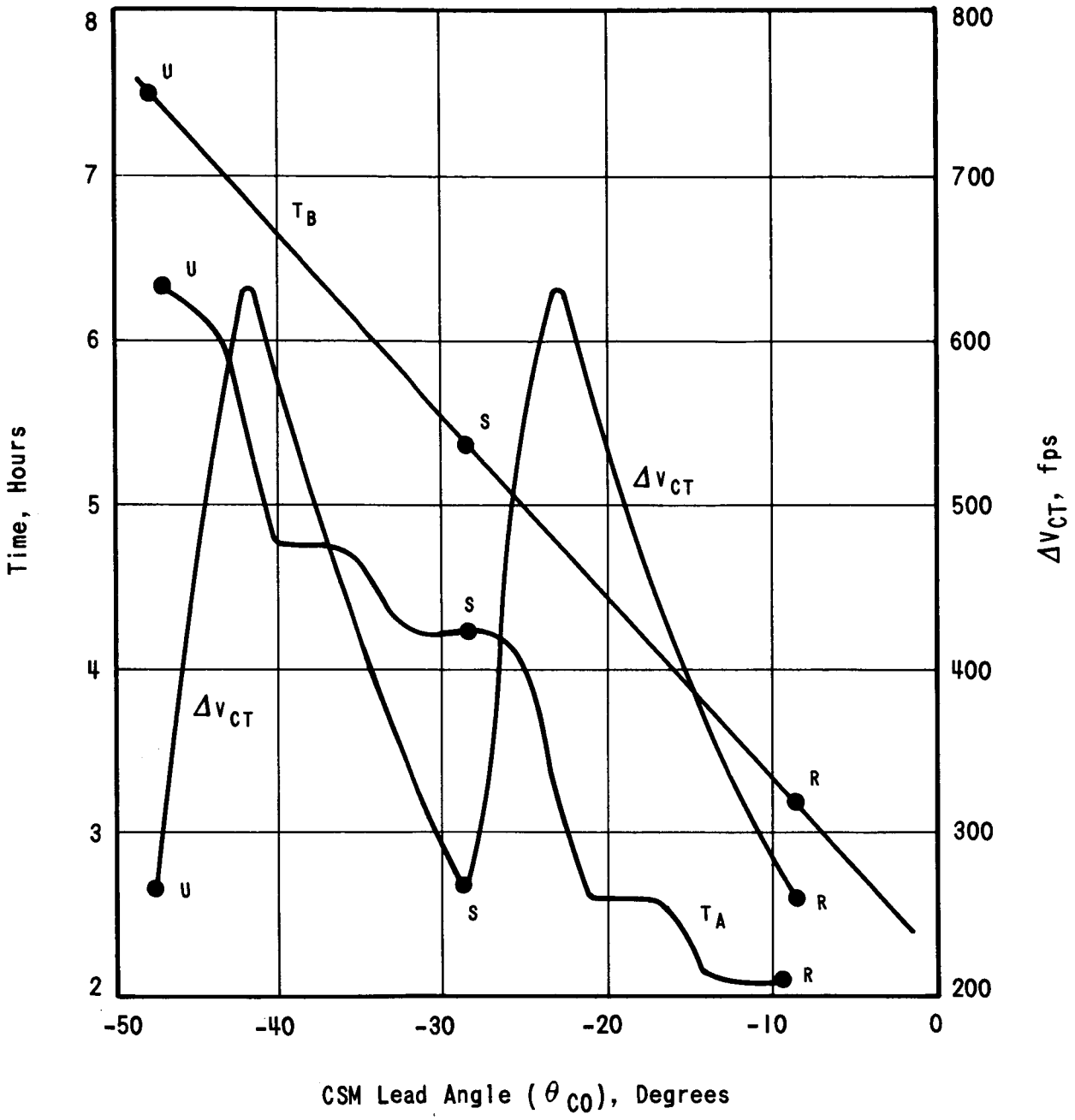

FIGURE 21 NON-COLLINEAR TRANSFER ORBITS, ΔV_{LO} = 273 fps
(T_B NOT OPTIMIZED)